# Mid-semester Examination

Physics IV,
B. Math. Year III,
January - May 2020,

February $26^{\text{th}}$, 2020.
10:00 am - 1:00 pm

This is a closed-book, closed-notes examination. You are allowed 1 page (two-sided) of handwritten/printed notes. Calculators are allowed; however, access to internet connection and/or files saved in mobile phones or computers are prohibited in any form. If values of fundamental constants are required, they will be provided (in SI units). The test is out of **50** points. Please answer all questions.

1. (a) State the two postulates of the special theory of relativity. [2.5+2.5]
   (b) Show that, for a particle of rest mass $m_0$ moving with a speed $u \ll c$, its kinetic energy $K$ will always be much smaller than its rest energy, $m_0c^2$. [5]
2. Imagine two rockets, A and B, both of proper length $L_0 = 1$m approach each other. Each rocket sees the other with approaching with speed $u < c$. Imagine the path of the rockets are shifted a tiny bit in the y-direction so that they don't collide. At the time when an observer on A sees the tip of A pass the tail of B, he fires a missile from A's tail towards B. According to the observer on A, however, the missile will miss because length of B is shorter than 1m, and therefore B's tip does not quite reach the tail of A. According an observer on B, however, the missile will hit, because the length of A is shorter than 1m, and therefore the tail of A falls within the length of B. Explain, from the viewpoint of the rest frame of *each* rocket, whether the missile will hit or miss i.e., explain which observer is right and why. (*Hint: It is best to analyze this situation by breaking up the situation into constituent events and analyze each event by turn.*) [5+5]
3. (a) Two rockets, with rest length $L_0$, are approaching the earth from opposite sides. Each has speed $|\vec{v}| = \frac{c}{2}$. How long does one rocket look to the other? [2]
   (b) A body quadruples its momentum when its speed doubles. What was its initial speed in units of $c$, i.e., what was $\frac{v_{\text{init}}}{c}$? [2]

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(c) A body of rest mass $m_0$ approaches another identical body initially at rest. After the collision, they stick together. What is the mass and momentum of the combined body after the collision? [3]

(d) A body of rest mass $m_0$ approaches an identical body at rest with speed $v$ in a reference frame S. What is the speed of the frame S' with respect to S such that the net momentum in S' is zero? Show that this reduces to the classical result if $v \ll c$. [3]

4. Show that the photon $\gamma$, with rest mass zero, cannot decay into an electron ($e^-$) and a positron ($e^+$) spontaneously i.e., the particle-physics process $\gamma \rightarrow e^- + e^+$, is relativistically forbidden (the symbol $\rightarrow$ here signifies "*decays into*"). For the purpose of this problem, you should consider the electron and the positron to be identical particles with same rest masses $m_0$, and the energy $E_\gamma$ of the photon to be related to its momentum by $E_\gamma = p_\gamma c$. (*Hint: Consider the conservation of energy and momentum of the process and arrive at a contradiction. Remember, if a process can be shown to be forbidden in one frame it is forbidden in any reference frame.*) [10]

5. (a) Consider a neutral particle, known as pi-meson, $\pi_0$, which, in its own rest frame, has a lifetime of $\tau_{\pi_0} = 26 \times 10^{-9}$ seconds. In the earth frame, it moves with a speed of $v = 0.95c$, $c = 3.0 \times 10^8$ m/s.

   i. What is its lifetime as seen in the laboratory at rest in the earth frame? [2.5]

   ii. What is the average distance it travels in the earth frame before decaying? [2.5]

   (b) The earth and the sun are 8.3 light-minutes apart (a light-minute is the distance light travels in a minute in vacuum). Ignore their relative motion in this problem and imagine that they are both at rest in a single inertial frame S. Suppose an event A happens on earth at $t = 0$ and another event B on the sun at $t = 2$ minutes in S. Find the time-difference between the two events A and B:

   i. for an observer who is traveling from the earth to the sun at a speed of $0.8c$? [2.5]

   ii. for an observer who is traveling in the opposite direction with the same speed? [2.5]